

A Theoretical Diagnosis on Light Speed Anisotropy from GRAAL Experiment

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The light speed anisotropy, i.e., the variation of the light speed with respect to direction in an “absolute” reference frame, is a profound issue in physics. The one-way experiment, performed at the GRAAL facility of the European Synchrotron Radiation Facility (ESRF) in Grenoble, reported results on the light speed anisotropy by Compton scattering of laser photons on high-energy electrons. We show in this paper that the azimuthal distribution of the GRAAL experiment data can be elegantly reproduced by a new theory of Lorentz invariance violation or space-time anisotropy, based on a general principle of physical independence of the mathematical background manifold.

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Isotropy and constancy of light speed are two basic properties of light in modern physics. Any evidence for their variation, even very tiny, will have profound implication in science. The anisotropy of the light speed in vacuum has been studied for more than 100 years, and the most famous exploration is the Michelson-Morley experiment in 1887 [1]. There are many modern analogies of experiments for the same purpose and most of them adopted round-trip or two-way path propagation of light involving averaged light speed. Therefore one-way experiments, which are sensitive to the first order of light speed variation, deserve particular attention. In this paper we provide a theoretical analysis of the results from the one-way experiment [2–5] performed at the GRAAL facility of the European Synchrotron Radiation Facility (ESRF) in Grenoble. We show that the available GRAAL results of azimuthal distribution can be explained by the light speed anisotropy suggested by a new theory [6] of the Lorentz invariance Violation (LV) or space-time anisotropy.

We provide a brief review on the principle [7] of the GRAAL experiment [2–5], in which the highly monochromatic electrons are scattered on the laser photons, for the study of light speed anisotropy in the “absolute” inertial frame at rest defined by null dipole of the Cosmic Microwave Background (CMB) radiation. In the head-on Compton scattering of the ultra-energy electrons and the low energy photons, the energy E of the scattered photon is given by

$$E = \frac{4\gamma^2 E_0}{1 + 4\gamma E_0/m_e + \theta^2 \gamma^2}, \quad (1)$$

where θ is the angle between the scattered photon and the incident electron, E_0 is the energy of the incident photon, and m_e and γ are the mass and Lorentz factor of the incident electron. On the other hand, the scattered electrons will separate from the main incident electrons beam, and there is a distance x between these two trajectories. The energy of the scattered photon can also be

written as

$$E = \frac{E_e x}{A + x}, \quad (2)$$

with E_e being the energy of the incident electron and A being a constant related with the experiment set-up. The maximum energy E of the Compton scattered photons is called as the Compton Edge (CE). From Eq.(1), CE can be obtained for $\theta = 0$. In this case, using Eqs. (1) and (2), we get

$$x_{\text{CE}} = \frac{4A\gamma E_0}{m_e}. \quad (3)$$

So

$$\delta x_{\text{CE}} = \frac{4AE_0}{m_e} \delta\gamma = -\frac{4AE_0}{m_e} \beta^2 \gamma^3 \delta c, \quad (4)$$

where $\gamma = (1 - \beta^2)^{-1/2}$ is the Lorentz factor of the incident electron. At the GRAAL facility, the mean energy E_e of the electron beam is 6.04 GeV, i.e., $\gamma = 11820$; three UV laser lines around 351 nm and a green line at 512 nm are used; and $A = 159.28 \pm 0.2$ mm.

Now, x_{CE} has been measured in the GRAAL experiment [2–5]. With the given stable energies E_0 and E_e , x_{CE} varies over different directions in the space when δc is azimuthal dependent. The azimuthal distribution for x_{CE} measured in the experiment determines the light speed anisotropy δc . The GRAAL results for the measured distance δx_{CE} related with the CE are shown in Fig. 1 for the data of the years 1998–2005 [3] and in Fig. 2 for the data in the year 2008 [4], revealing the robust CE azimuthal variation. The limits on the light speed anisotropy are reported in Ref. [5], in which the azimuthal distribution presented in Figs. 1, 2 was not discussed. We show in this paper that the GRAAL results in Figs. 1, 2 can be elegantly reproduced by our new theory of Lorentz violation [6], briefly illustrated below for the free photon sector.

Nowadays, Lorentz invariance violation has triggered more and more interests in physics (see, e.g., Refs. [8–12] and references therein). Our framework [6] is a new

fundamental theory of Lorentz invariance violation from basic principles instead of from phenomenological considerations. We proposed a general principle of physical independence of the mathematical background manifold, and based on such principle we revealed the replacements $\partial^\alpha \rightarrow M^{\alpha\beta} \partial_\beta$ and $D^\alpha \rightarrow M^{\alpha\beta} D_\beta$ for the ordinary partial ∂_α and the covariant derivative D_α . $M^{\alpha\beta}$ is a local matrix called Background Matrix (BM) and can be divided into the sum of two matrices, i.e., $M^{\alpha\beta} = g^{\alpha\beta} + \Delta^{\alpha\beta}$, where $g^{\alpha\beta}$ is the metric of space-time and $\Delta^{\alpha\beta}$ is a new matrix which brings new terms violating Lorentz invariance in the standard model, therefore we denote the new framework as the Standard Model Supplement (SMS). Then the Lagrangian for the free gauge particle photon reads

$$\mathcal{L}_G = -\frac{1}{4} F^{\alpha\beta} F_{\alpha\beta} - F_{\mu\nu} \Delta^{\mu\alpha} \partial_\alpha A^\nu - \frac{1}{2} \Delta^{\alpha\beta} \Delta^{\mu\nu} (g_{\alpha\mu} \partial_\beta A^\rho \partial_\nu A_\rho - \partial_\beta A_\mu \partial_\nu A_\alpha). \quad (5)$$

Since $\Delta^{\alpha\beta}$ contains all the LV information for the space-time, we call it Lorentz invariance Violation Matrix (LVM). All the LV effects vanish when $\Delta^{\alpha\beta} = 0$. More details of this new framework can be found in Ref. [6].

We thus get the modified Maxwell equation (or motion equation)

$$\Pi^{\gamma\rho} A_\rho = 0, \quad (6)$$

where $\Pi^{\gamma\rho}$ is also the inverse of the photon propagator

$$\Pi^{\gamma\rho} = -g^{\gamma\rho} \partial^2 + \partial^\gamma \partial^\rho - 2g^{\gamma\rho} \Delta^{\mu\alpha} \partial_\mu \partial_\alpha + 2\Delta^{\gamma\alpha} \partial^\rho \partial_\alpha + \Delta^{\gamma\beta} \Delta^{\rho\nu} \partial_\beta \partial_\nu - g^{\gamma\rho} g_{\alpha\mu} \Delta^{\alpha\beta} \Delta^{\mu\nu} \partial_\beta \partial_\nu. \quad (7)$$

With the Fourier decomposition $A_\rho = \int dp A_\rho(p) e^{-ip \cdot x}$ and the Lorentz gauge condition $\partial^\alpha A_\alpha = 0$ for the gauge field, we can re-write Eq. (6) as

$$\Pi^{\gamma\rho}(p) A_\rho(p) = 0,$$

where

$$\Pi^{\gamma\rho}(p) = g^{\gamma\rho} (p^2 + g_{\alpha\mu} \Delta^{\alpha\beta} \Delta^{\mu\nu} p_\beta p_\nu + 2\Delta^{\alpha\beta} p_\alpha p_\beta) - \Delta^{\gamma\beta} \Delta^{\rho\nu} p_\beta p_\nu,$$

which is the inverse of the free photon propagator in the momentum space. A general parameterization for p_α can be done with spherical coordinates, so p_α can be expressed as $(E, -|\vec{p}| \sin \theta \cos \phi, -|\vec{p}| \sin \theta \sin \phi, -|\vec{p}| \cos \theta)$, where the light speed $c = 1$. We find that there is a zero eigenvalue and a corresponding eigenvector $A_\rho(p)$ for the matrix $\Pi^{\gamma\rho}(p)$. So the determinant must be zero for the existence of the solution $A_\rho(p)$

$$\det(\Pi^{\gamma\rho}(p)) = 0. \quad (8)$$

Then we have the equation

$$\sum_{i=0}^8 \lambda_i(\Delta^{\alpha\beta}, \theta, \phi) E^i |\vec{p}|^{8-i} = 0.$$

The coefficient $\lambda_i(\Delta^{\alpha\beta}, \theta, \phi)$ is a variable related to the LVM $\Delta^{\alpha\beta}$ and the angles θ and ϕ . So there are 8 real solutions for $E(|\vec{p}|)$ at most, and in general there are no analytical solutions for a general high order linear equation. But for some simple cases of the LVM $\Delta^{\alpha\beta}$, we expect some analytical solutions for E . Anyway, E can be solved formally as $E = f_i(\Delta^{\alpha\beta}, \theta, \phi) |\vec{p}|$, for $i = 1 \dots N$, and $1 \leq N \leq 8$. $f_i(\Delta^{\alpha\beta}, \theta, \phi)$ is a real variable and is independent of the momentum magnitude $|\vec{p}|$ because the photon is massless in the Lagrangian of Eq. (5). So the free photon velocity is

$$c_{\gamma i} \equiv \frac{dE}{d|\vec{p}|} = f_i(\Delta^{\alpha\beta}, \theta, \phi), \quad \text{for } i = 1 \dots N, \quad 1 \leq N \leq 8, \quad (9)$$

which means: i) The free photon propagates in the space with at most 8 group velocities; ii) For each mode, the light speed $c_{\gamma i}$ might be azimuthal dependent and not a constant. As we have known, the light spreads with different group velocities for different directions in the anisotropic media in optics. In analogy, we may view the space-time as a kind of media intuitively. However, there are essential differences between the optical case and the photon case here, because all the consequences of the N modes and the light speed anisotropy are results from the Lorentz invariance violation or the space-time anisotropy suggested by the new framework.

We need to clarify some essential points concerning the light speeds, i.e., c_γ in our work and the conventional light speed constant c . c_γ is determined by the Maxwell equations or the propagator in QED, and represents the real propagation speed of the photon or the Electromagnetic wave freely propagating in the space-time, whereas c is related with the Lorentz group and the space-time metric, and serves as a constant. These two speeds are regarded as the same thing generally, but we should make clear that they are two different concepts. In the natural units, $c = 1$. When we write it explicitly in any unit system, the metric is $g_{\alpha\beta} = \text{diag}(1, -1/c^2, -1/c^2, -1/c^2)$, so $g^{\alpha\beta} = \text{diag}(1, -c^2, -c^2, -c^2)$. We see that the light speed c is related with the unit definitions of the time and the space. And an element $R^{\alpha\beta}$ of the Lorentz group is defined as the one which satisfies $g_{\beta\nu} R^{\alpha\beta} R^{\mu\nu} = g^{\alpha\mu}$ where c is invariant, so we call c Lorentz invariant constant. In this article, we do not consider the light speed c in our derivation, i.e., we set $c = 1$ in the natural units. Instead, the light speed implied in our arguments is actually the propagating velocity c_γ of the photon or the Electromagnetic wave, and generally $c_\gamma \neq c$ here.

We show now that our theory suggests the light speed anisotropy with respect to the azimuthal angle in an ‘‘absolute’’ reference frame. To understand the azimuthal distribution of the GRAAL data, let us consider two simple forms of $\Delta^{\alpha\beta}$. One is

$$\Delta^{\alpha\beta} = \xi m^\alpha n^\beta, \quad (10)$$

where m and n are two unit vectors in the space-time and ξ measures the magnitude of LV. When n and m are parallel, $\Delta^{\alpha\beta}$ of Eq. (10) represents that there exists a strain along the direction n in the space-time. This case of the LVM can help us to check whether there is a preferred direction n in the space-time. When n and m are orthogonal, Eq. (10) represents a shear in the plane spanned by the two vectors m and n [13]. Another useful parameterization for $\Delta^{\alpha\beta}$ is

$$\Delta^{\alpha\beta} = \lambda k^\alpha, \quad k^2 = \pm 1, \quad (11)$$

which represents a translation along a direction k and λ measures the magnitude of LV too. Timelike unit vectors can be parameterized as $(\cosh \zeta, \sinh \zeta \sin \theta \cos \phi, \sinh \zeta \sin \theta \sin \phi, \sinh \zeta \cos \theta)$, while spacelike ones as $(\sinh \zeta, \cosh \zeta \sin \theta \cos \phi, \cosh \zeta \sin \theta \sin \phi, \cosh \zeta \cos \theta)$, where ζ , θ and ϕ are three variables to parameterize the unit vectors.

Now, we assume that there is a preferred direction $n = m$ for the space-time. For the sake of generality, we can take this direction as the x -axis, i.e., $\zeta = 0$, $\theta = \pi/2$ and $\phi = 0$. So Eq. (10) reads $\Delta^{\alpha\beta} = \text{diag}(0, \xi, 0, 0)$, which is substituted into Eq. (8) and then we can obtain all the two physical solutions for the light speed $c_{\gamma i}$. $c_{\gamma 1} = \sqrt{1 - (2\xi - \xi^2) \sin^2 \theta \cos^2 \phi}$, $c_{\gamma 2} = \sqrt{1 - 2\xi \sin^2 \theta \cos^2 \phi}$. Neglecting the higher powers of ξ , we can get $\delta c_{\gamma a}/c_\gamma \equiv |c_{\gamma \max} - c_{\gamma \min}|/c_\gamma \propto |\xi|$ and $\delta c_{\gamma m}/c_\gamma \equiv |c_{\gamma 1} - c_{\gamma 2}|/c_\gamma \propto \xi^2$. $\delta c_{\gamma a}$ and $\delta c_{\gamma m}$ represent the differences resulting from the angular distribution and the mode differences respectively. So we find two interesting results: i) The light speed difference between two modes is proportional to the square of the element of the LVM, i.e. $\delta c_{\gamma m}/c_\gamma \propto \xi^2$; ii) For each mode, the light speed may be direction dependent, and this anisotropy is linearly proportional to the element of the LVM, i.e. $\delta c_{\gamma a}/c_\gamma \propto |\xi|$. When $\xi = 0$, the light speed $c_{\gamma i}$ is equal to the constant $c = 1$, and the angle distribution of c_γ is a sphere of radius 1 in the space. But it is direction dependent now for $\xi \neq 0$. Along the direction n , the light speed decreases ($\xi > 0$) or increases ($\xi < 0$), and the distribution for c_γ is not spherical any more.

For $\Delta^{\alpha\beta}$, the sum of the two above cases reads

$$\Delta^{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \xi & 0 & 0 \\ \lambda & \lambda & \lambda & \lambda \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (12)$$

which means $n = m = (0, 1, 0, 0)$ in Eq. (10) and $k = (0, 0, 1, 0)$ in Eq. (11). This $\Delta^{\alpha\beta}$ represents that there is a

preferred direction $n = (0, 1, 0, 0)$ and a translation along $k = (0, 0, 1, 0)$ for the space-time, meaning the space-time is not isotropic now. The equation (8) is so complicated that we can hardly solve all the eight analytical solutions for the light speed in Eq. (9). We get two solutions. One

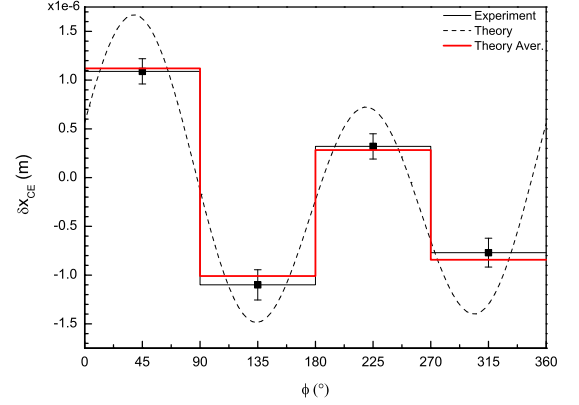


FIG. 1. δx_{CE} azimuthal distribution vs angles of the GRAAL data of the years 1998-2005 on a plane (x - y plane or $\theta = \pi/2$). $\xi = -2.89 \times 10^{-13}$, $\lambda = 6.53 \times 10^{-14}$.

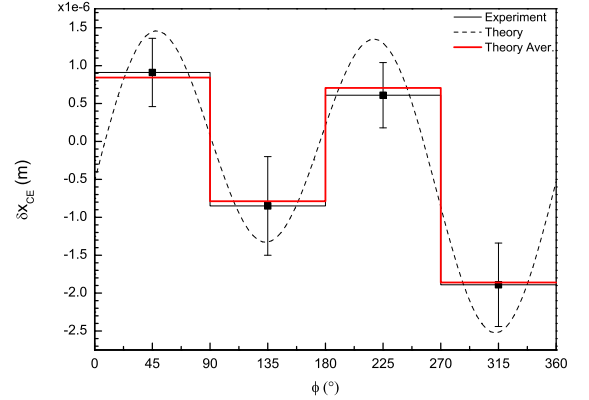


FIG. 2. δx_{CE} azimuthal distribution vs angles of the GRAAL data of the year 2008 on a plane (x - y plane or $\theta = \pi/2$). $\xi = -3.64 \times 10^{-13}$, $\lambda = 8.24 \times 10^{-14}$.

of them is physical and its explicit form is also lengthy

$$c_\gamma = \frac{[\sin \theta (\sin \phi + \cos \phi) + \cos \theta] \lambda^2 - \sin \theta \sin \phi \lambda - \sqrt{h}}{-1 + \lambda^2} \quad (13)$$

with

$$h = 1 + [2 \sin^2 \theta (\cos^2 \phi - \sin \phi \cos \phi - 1) - 2 \sin \theta \cos \theta \sin \phi] \lambda + [\sin^2 \theta \sin \phi (\sin \phi + 2 \cos \phi) + 2 \sin \theta \cos \theta (\sin \phi + \cos \phi)] \lambda^2 + (-1 + \lambda^2) (2\xi - \xi^2) \sin^2 \theta \cos^2 \phi.$$

TABLE I. Constraints on the element ξ of the LVM from some light speed anisotropy experiments.

Experiment	$\delta c_{\gamma a}/c_{\gamma}, \xi $
Refs. [2, 3]	3×10^{-12} one-way
Ref. [4]	1.0×10^{-14} one-way
Refs. [14, 15]	3×10^{-9} one-way
Ref. [16]	3.5×10^{-7} one-way
Refs. [17, 18] (cf. [19, 20])	3×10^{-17} two-way

Finally, Eq. (4) becomes

$$\delta x_{\text{CE}} = -\frac{4AE_0}{m_e} \beta^2 \gamma^3 (c_{\gamma} - c'), \quad (14)$$

where c' is an effective constant and $c' \rightarrow 1$. The light speed c_{γ} is the specific form of Eq. (13). Thus we can compare our theoretical calculations of the light speed anisotropy with the experimental results presented in Figs. 1 and 2, in which the solid curves represent the GRAAL data from Refs. [3, 4]. The dashed curves are the calculated results of Eq. (14), and they are obtained to fit the experimental curves. The bright-color solid curves are the calculated results averaged over 90 degrees to fit the experimental curves too, and they are completely allowed within error bars by the GRAAL data. In Fig. 1, the best fit parameters are: $\xi = -2.89 \times 10^{-13}$, $\lambda = 6.53 \times 10^{-14}$, and in Fig. 2, $\xi = -3.64 \times 10^{-13}$, $\lambda = 8.24 \times 10^{-14}$. We also find that the best fit occurs when $\xi \simeq -4\lambda$. In this article, we can take the average and get $\xi = -3 \times 10^{-13}$ and $\lambda = 7 \times 10^{-14}$. So the LVM for photons can be approximated by

$$\Delta^{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -3 \times 10^{-13} & 0 & 0 \\ 7 \times 10^{-14} & 7 \times 10^{-14} & 7 \times 10^{-14} & 7 \times 10^{-14} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and $\delta c_{\gamma a}/c_{\gamma} \simeq 10^{-14}$ - 10^{-13} . For comparison, the constraints on the element ξ of the LVM are given in Tab. I from some other experimental results on light speed anisotropy.

The GRAAL data shown in Figs. 1 and 2 manifest a consistency between the two periods of experiment of the years 1998-2005 and 2008 [3, 4], but such data were not discussed in a recent publication [5], where only limits of the order of 10^{-14} on parameters related to light speed anisotropy were reported. We may consider our work as providing the constraints on the specific photon LVM of Eq. (12) to the order of 10^{-14} . Though the regularity revealed by the GRAAL data in Figs. 1 and 2 needs to be further confirmed by future experiments,

it is a surprise that our simple model calculation can successfully reproduce the azimuthal distribution of the reported GRAAL data in an elegant manner. Therefore we may also consider our work as giving an interpretation of the light speed azimuthal distribution reported in the GRAAL experiment. In our framework, the Lorentz invariance violation or the space-time anisotropy for the photon is the source for the light speed anisotropy shown in the GRAAL data.

We thus conclude that we provide a novel explanation for the light speed anisotropy in the GRAAL experiment, based on a new fundamental theory of Lorentz invariance violation or space-time anisotropy. This work not only manifests the elegant application of the new theory to fit experimental results, but also suggests new chances to test the theoretical predictions from the new framework and to constraint the newly introduced Lorentz invariance violation matrix by future experiments.

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